Band Gap Energies of Silicon and Germanium

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Measurements of voltage and current through germanium and silicon diodes were made at thermal equilibrium over a broad range of temperatures. Applying these data to our theory allowed for the calculation of the band gap energies of silicon and germanium. The obtained values are $E_g(Si) = (1.01 \pm 0.19) \text{ eV (95\% CI)}$ and $E_g(Ge) = (0.68 \pm 0.15) \text{ eV (95\% CI)}$, which are both in agreement with the accepted values of $E_g(Si) = 1.17 \text{ eV}$ and $E_g(Ge) = 0.74 \text{ eV}$ [1].

Solid state physics has many useful applications to everyday life. Perhaps some of the most ubiquitous materials found in this field are semiconductors. Nearly all modern-day electronics, such as computers and cell phones, use semiconductors. As such, knowing how semiconductors work and why they are used in electronics will provide us with a good basic understanding of one of today’s most prevalent technologies: diodes.

A diode is essentially a one-way gate in a circuit. It is a $pn$-junction made of a semiconductor. The $p$-type (positive) semiconductor can be thought of as having positive “holes,” which electrons from the $n$-type (negative) semiconductor try to fill [1]. Due to the polarity of the diode, there is a bias associated with it. If we insert such a diode into a circuit so that the bias is a reverse bias, we then have no flow through the diode; the one-way direction of the diode is pointed the wrong way, so no current passes through it. However, the material that makes up the diode is a semiconductor. So, if we supply the electrons in the diode with enough energy through an electric field, we can cause the material to conduct, thus restoring the current through the circuit. The minimum amount of energy needed to have the electrons leave the valence band of their atoms to join the conduction band of flowing electrons is called the band gap energy, $E_g$ [1]. By building a circuit that takes advantage of the properties of the diodes, we can determine what $E_g$ is for a given diode and its semiconducting material. In our experiment, we will determine the band gap energies for silicon (Si) and germanium (Ge), two common semiconductors. The diodes we are using are 1N34A from Microsemi for Ge and 1N4001 from Fairchild for Si.

Due to the thermal dependence of the electron flow, we can use Maxwell-Boltzmann statistics to describe the $I-V$ characteristics of the diode. We find

$$I = I_0(e^{eV/kT} - 1)$$ (1)

where $I_0$ is the current through the diode when there is no bias applied to it (also known as the reverse saturation current [2]), $I$ is the applied current, $V$ is the applied voltage, $e$ is elementary charge, $k$ is the Boltzmann constant, and $T$ is the temperature at which the system is
in thermal equilibrium [1]. Though this is a good general approximation of a pn-junction diode, more can be done to simulate the real-world features of the diode.

For all diodes, there is a parameter $h$ which is characteristic of each specific device. Thankfully, $h$ can be easily determined, as we will see soon. Another factor we must consider is the leakage current $V_0/R$ of the diode, for the voltage $V_0$ of the diode when non-biased and resistance $R$ of the diode. Ideally, this factor would be zero, but real diodes have a small resistance associated with them. As with $h$, we will treat $V_0/R$ as a parameter as it is not something that we are interested in exploring deeply; it is just that both terms must be considered if we want our model to be as effective as possible. Adding $h$ and $V_0/R$ into Eq. (1) gives

$$I = I_0(e^{V/hkT} - 1) - V_0/R,$$

which we will use as our theoretical model. In Eq. (2), we see that we need to know $I$ and $V$ in order to determine $I_0$. It is $I_0$, the reverse saturation current of the diode, that allows us to calculate $E_g$. We know that $E = qV$ in general, which in this case implies that $E_g = eV$. Also, $I_0$ and $E_g$ are related by

$$\ln(I_0) = \ln(BT^{2/3}) - (E_g/hk)(1/T)$$

for parameter $B$. However, the first term is negligible over the temperature range we will run our experiment in [2], so we simplify Eq. (3) as

$$\ln(I_0) = (-E_g/hk)(1/T).$$

In order to apply our models from Eq. (2) and Eq. (4), we first need to record data for current $I$ and voltage $V$ while the diode is at thermal equilibrium. In our circuit

![Circuit Diagram](image)

**FIG. 1:** In our circuit, we have placed the diode so that we can measure the voltage through the circuit both with ($V_1$) and without ($V_2$) the diode. This allows for a calculation of the voltage drop ($V = V_2 - V_1$) across the diode with an oscilloscope whose data is retrieved with LabPro software. By inserting a resistor $R$ after the diode, we can also calculate the current $I$ through the diode since $I = V_1/R$. Having $V$ and $I$ for a given $T$ allows us to find $I_0$. 
diagram, we have an oscilloscope (Tektronix TDS 2012B DSO) set up to record the voltages at two different points in the circuit: one point where the current does not go through the diode ($V_2$), and one point where it does ($V_1$), as seen in Fig. (1). By doing this, we can calculate the drop in voltage $V$ across the diode seen in our above equations. Also, since we already have the oscilloscope measuring voltage, we put a resistor of known resistance $R = (997 \pm 1) \, \Omega$ in the circuit where the diode is present so a calculation can be made for the current $I$ through the diode from the basic relation $I = V_1 / R$. Separate yet identical circuits were built for both the Si and Ge diodes.

In order to get as much data as possible, we use a function generator to create a triangle voltage function for the circuit. This is an advantageous function since it allows us to apply an increasing voltage, drop that voltage, and then repeat. We use the oscilloscope to average 128 readings so that we have stable values for the voltages. LabPro software is used to extract data from the oscilloscopes so that it can be analyzed. Then, we plot $I$ verses $V$ so that we can fit Eq. (2) to our data. Fig. (2) is a representative $IV$-curve from our experiment. Curves are fit to our data at about 30 different temperatures for each diode. Thus, we have about 30 different $I_0$ values at different temperatures $T$ for each diode. Also, we set $e/hk$ as a parameter in our fit, allowing us to calculate $h$ for both diodes. We get $h_{Si} = 1.697$ and $h_{Ge} = 4.221$, which are both on the correct order of magnitude for general $h$ values.

![Image of IV curve](image_url)

**FIG. 2:** Here is an $IV$-plot for Si at $T = 19.6 \, ^\circ C$. For this example, fitting Eq. (2) to the data yields $I_0 = (58.57 \pm 7.18) \times 10^{-11} \, A$ (95% CI). Using these $T$ and $I_0$ values with other such values for Si allows for the calculation of $E_g$ for Si. The same process is used for Ge.

With our $I_0$ values extracted, we then plot $\ln(I_0)$ against $(1/T)$ to determine $E_g$ for Si and Ge. However, we will flip the axes since our uncertainty in temperature was far greater than that in current $I_0$; our software bet-
FIG. 3: By recording the value reported by our thermocouples \((T_{\text{measured}})\) with the known temperature \((T_{\text{actual}})\) and doing a linear fit, we are able to calibrate our temperatures. There is a small but noticeable curvature to the data, which may be due to the low number of data points.

ter handles uncertainty associated with the vertical axis variable. This must be done because of our inability to easily calibrate our temperature values during thermal equilibrium (Fig. (3)). To measure \(T\), we use thermocouples that are epoxied to our diodes. LabPro lets us read out the temperatures from the thermocouples. The software also has the thermocouples make multiple measurements per minute so that we know if the diodes are or are not in thermal equilibrium. Taking data with temperatures ranging from as cold as liquid nitrogen (77 K) to as hot as near boiling water (360 K) helps ensure that our model is consistent over many temperatures. However, our thermocouples are uncalibrated. Since a regular mercury thermometer cannot withstand our entire wanted range of temperatures, we must use known values, such as liquid nitrogen, to calibrate the thermocouples. Some other known temperatures of use are 273 K in ice water and an acetone bath filled with dry ice at 263 K. In total, we have six known values with which to calibrate the thermocouples. Unfortunately, having so few data points causes a relatively large uncertainty of about 10\% in \(T\).

Now that we have \(I_0\) values for their respective \(T\) values and their uncertainties, we fit a line to the data and use the slope to find \(E_g\) (Fig. (4)). We obtain \(E_g(Si) = (1.01 \pm 0.19)\ \text{eV (95\% CI)}\) and \(E_g(Ge) = (0.68 \pm 0.15)\ \text{eV (95\% CI)}\) for our diodes, which are in good agreement with the accepted values of \(E_g(Si) = 1.17\ \text{eV}\) and \(E_g(Ge) = 0.74\ \text{eV [1]}\).

While our values for \(E_g\) for Si and Ge do agree with the accepted values, we see that perhaps our model could be improved. In both Si and Ge, there are noticeable curvatures to the data seen in Fig. (4). To try to account for this curvature, we decided to try and fit our proto-
model in Eq. (3) to the data to see if we should not have disregarded the first term with the $T^{2/3}$ dependence. Doing so resulted not only in slightly less accurate $E_g$ values, but also much larger uncertainties. So, we do believe that Eq. (4) is a better model for determining $E_g$.

Another possible reason for the curvature in Fig. (4) is our temperature calibration; it too has a visible curvature, as seen in Fig. (3). However, we do not know if this is an actual data trend, or if it is just the result of low statistics. Also, since the manufacturer of the thermocouples stated that a linear fit is appropriate for calibration, we are hesitant to simply try polynomial fits that will to match the data. The only real solution to this is to take more data points at known temperatures, which could be done on a hot plate with an electronically controlled temperature readout. That way, we could easily
use many temperatures for the calibration fit. By taking advantage of the IV characteristics of diodes, we calculated band gap energies of Si and Ge that are in agreement with the accepted values. Some improvements can be made to the experiment, mainly in temperature calibration, but our values show that the theory we used of how semiconductors relate to current, voltage, and temperature accurately models the data.