Measuring Mie Scattering from Ionized, Trapped Dielectric Particles

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Light scattered by particles with sizes on the order of the wavelength is described using Mie theory. Mie scattering is very sensitive to the scatterer’s size, as well as the scattering angle. As a result, Mie scattering can be used to measure the size of microspheres accurately. Previous undergraduate-level experiments have measured Mie scattering from microspheres suspended in liquid. Alternatively, we propose measuring Mie scattering from a single particle trapped in a double-needle Paul ion trap. We present preliminary results and the ion radius is estimated to be about 10 µm.
Light scatters whenever it encounters a physical object. Its scattering behavior depends on its wavelength, as well as the size of the object. For particles much larger than the light wavelength, geometric optics is a good approximation of the scattered field; for particles much smaller than the wavelength, one can use the Rayleigh scattering formula. However, for particles whose sizes are on the order of the wavelength, there is no simple approximation. Light scattering from such particles is described by the Mie solutions to Maxwell equations. The remarkable feature of Mie theory is that it predicts the angular dependence of scattered intensity, as illustrated in Fig. 1. Since this angular dependence is very sensitive to the particle size, Mie scattering has been used as an accurate sizing method of microspheres [1, 2]. In previous undergraduate-level experiments, particles suspended in liquid were used to perform Mie-scattering measurements, assuming that the light scattering behavior of the suspension is the same as that of a single particle [1, 3, 4]. Here we propose an experiment that allows Mie-scattering measurement from a single particle levitated in an electrodynamic trap. This is an improvement upon previous experiments in that we can measure the actual light scattering off an individual ion. Ion trapping is based on the principle that an alternating electric field can trap an ion. Previous work has been devoted to developing ion traps [5–7].

![Diagram of wavefronts in Mie scattering](image)

**FIG. 1.** Illustration of wavefronts in Mie scattering. $N$ and $N_1$ are the indices of refraction of ambient medium and the scatterer, respectively. $\lambda$ is the wavelength of incident light. Scattering intensity depends on the angle $\theta$ [1].

The derivation of Mie theory is mathematically advanced and can be found in the liter-
ature [8, 9]. Essentially, Mie scattering depends on the size parameter $x$, which is defined as

$$x = \frac{2\pi Na}{\lambda},$$

where $N$ is the index of refraction of the surrounding medium, $a$ is the radius of the sphere and $\lambda$ is the wavelength of incident light in vacuum. It is interesting to note that Mie scattering depends on the ratio of $a/\lambda$ rather than solely on radius or wavelength. Fig. (2) shows calculated angular scattering distribution for $x = 5, 10, \text{ and } 20$ [2]. As $x$ increases, oscillations in the angular distribution curve becomes more prominent. For our experiment, we used $\lambda = 532 \text{ nm laser}$, an average particle radius of $a = 15 \mu\text{m}$, and, assuming a reflective index of refraction of air $N = 1$, we have $x = 177$.

![Graphs showing angular scattering distributions](image)

**FIG. 2.** Calculated angular scattering distributions with (a) $x = 5$, (b) $x = 10$, and (c) $x = 20$ for differently polarized light [1].

During the course of developing this project, we realized that a double-needle electrodynamic trap is ideal for this measurement due to its simplicity. We constructed a double-needle trap. The distance between the two Tungsten needles is $1.56 \pm 0.12 \text{ mm}$, while the distance between the two stainless steel cylinders is $4.60 \pm 0.03 \text{ mm}$. The needles have a
0.62±0.01 mm diameter; the cylinders have a inner diameter of 1.73±0.02 mm and an outer diameter of 2.43±0.01 mm. The needles and the cylinders are separated by a ceramic tube. The schematic is shown in Fig. 3. The radiofrequency (RF) voltage on the trap electrodes are provided by a microwave transformer; the input voltage into the microwave transformer is 50 V at 60 Hz. The voltage on the needles are \( V_{\text{trap}} = 845.0 \pm 0.5 \, \text{V}_{\text{rms}} \) (95% CI). The trap is placed in an acrylic box which provides shielding against air currents. The lower part of the trap is fixed onto the optical table via a post. The upper trap is connected to a three-dimensional translational stage, which allows fine adjustments of the relative position between the two electrodes.

![Schematic of the double-needle trap](image)

**Units:** mm (95% CI)

- GND: 4.60±0.03 mm
- RF: 1.56±0.12 mm
- Ceramics: 1.73±0.02 mm
- Stainless steel: 2.43±0.01 mm
- Tungsten: 0.62±0.01 mm

**FIG. 3.** Schematics of the double-needle trap. The metal needle is placed inside a metal tube, with a ceramic insulating tube in between (not shown in graph). A RF high voltage is applied to the needles, while the tubes are grounded. Dimensions of the key parts are labeled as above.

Since the acrylic box needs to be enclosed to perform scattering measurements, we built a robotic arm with LEGO components to perform the sweep automatically. The arm is driven by a pair of gears with ratio 7 : 1. The motor is controlled by the NXT (1.0) controller, whose internal sensor gives digital feedback on the angular location of the motor in arbitrary units. We mounted a photomultiplier tube (PMT) at the end of the arm. The distance between the ion and the PMT is 30.5 ± 0.4 cm (95% CI). To collect more light, a lens with focal length \( f = 3.81 \, \text{cm} \) was mounted in front of the PMT via a lens tube, 22.6 ± 0.4 cm (95% CI) away from the trap center, shown in Fig. 4. We inserted two neutral density filters with total attenuation 7.0 at \( \theta = 0^\circ \) to prevent the laser from saturating the PMT. Input voltage for the PMT was 460 V. A National Instruments USB-6009 computer data acquisition board
digitalized the analog signal from the PMT through a LM741 inverting unity amplifier with $R_1 = R_2 = 510$ kΩ. Under these conditions, background signal level was about 0.2 V. We used a single LabVIEW program to control the NXT and data acquisition from the PMT.

![Diagram of the experimental setup](image)

**FIG. 4.** A photomultiplier tube (PMT) is mounted at the end of the LEGO arm, whose motor is placed below the double-needle trap. Two lasers were used to characterize the sweeping motion of the arm. Laser 1 was turned off during actual scattering light measurements.

Two lasers were used to characterize the motion of the LEGO arm. Both lasers were adjusted so that they illuminated the ion in the trap while their beams were parallel with the table. Angular separation of the two beams was $\Delta \theta = 90.0^\circ \pm 1.1^\circ$ (95% CI). With both lasers turned on and no ion in the trap, light intensity was recorded as the PMT swept from approximately $\theta = 110^\circ$ to $\theta = -5^\circ$ continuously. Data collected from 7 trials were interpolated and summed to produce the points in Fig. 5 at an interval of 0.2 NXT ticks (arbitrary units). The points were fitted to Gaussian functions, which are the continuous curves in Fig. 5. By measuring the number of ticks between the two peaks, we found the corresponding average of $0.143^\circ \pm 0.002^\circ$ per tick (95% CI).

Microspheres were manually inserted into the trapping region using an ionizing plastic rod. The spheres were trapped by the RF potential and were loaded probabilistically. We found that trapped ions were extremely susceptible to air flow; we loaded with the acrylic box completely closed except for a small hole used to inset the rod. We first performed
FIG. 5. Data from seven sweeps were converted into seven interpolation functions in Mathematica. The points were generated using the sum of the interpolation functions, at a step size of 0.2. The points were fitted to a Gaussian function, which is shown as the curves.

ten measurements of the background, and then ten measurements with an ion in the trap, shown in Fig. 6 and Fig. 7, respectively.

The background data in Fig. 6 show that the background level is mostly low and constant when the PMT is not directly illuminated by the laser beam. In addition, aside from the horizontal shifts due to the difficulty in starting the PMT from the same location for each trial, the graphs are essentially the same. Looking at Fig. 7, we see that there are numerous oscillations in the scattered light intensity in the trials in with a loaded ion that is not present in the background intensity. We suspect that the oscillations are due to the constructive and destructive interference patterns that resulted from Mie scattering. The averages of the background and ion data were shown in Fig. 8. The scattered light intensity from the ion is significantly higher than background. The difference between the two is not isotropic, which qualitatively agrees with the Mie theory.

The small oscillations in our data have rather short period (compare with Fig. 2 in Ref. [1]), indicating that the ion has a large $x$. The approximated Mie scattering formula provided by Drake and Gordon was used to estimate our ion size [4]:

$$I(\theta) = I_0(3|J_1(\beta)|/\beta + \beta^{-3/2})^2(1 + \cos^2 \theta)/2,$$

(2)

where $I$ is the scattered intensity, $J_1(\beta) = (\sin \beta - \beta \cos \beta)/\beta^2$, and $\beta = ka(1+N^2-2N \cos \theta)$. 

FIG. 6. Ten sets of background data plotted with arbitrary vertical offsets. The main difference between each of the trials is horizontal translation of the peak. This is due to the difficulty in starting the detection arm at the same location each time. However, the background level is mostly low and constant, meaning there was little light scattering off the trap electrodes.

FIG. 7. Ten sets of data with ion in the trap plotted with arbitrary vertical offsets. At the top in red is the average of the ten trials. We can see that there are definite places of constructive and destructive interference due to the Mie scattering.
FIG. 8. Above are the average trials of the background light intensity and light reflecting off of an ion in the trap. As we can see, it is obvious that there is definite Mie scattering and definite places of constructive and destructive interference that is predicted by Mie theory that is not in the background light.

Numerical calculation for a particle with radius $a = 10 \, \mu m$ is shown in Fig. 9. Since the computed curve demonstrates oscillations with similar period as in our data, we estimate the ion radius to be about $10 \, \mu m$.

From the data we see that our PMT is detecting light that is reflecting off the ion and that it is nontrivial; we measured real Mie scattering. One improvement that will be made on the present apparatus is to attach the motor onto the end of the rotating arm connected to a wheel and have that turn the PMT. This should reduce the systematic errors in the ability to restart the PMT sensor in the same location after every trial. Additionally, another improvement that can be made is loading ions of known size into the double needle trap via electro-ionization spray. This would allow one to have a value for $x$, the ratio of the wavelength to particle radius, and compare the theory to the measured intensity of light. A further enhancement to our current design would be to place the entire apparatus in a vacuum chamber. This would help decrease the possibility of the ion being knocked out of
FIG. 9. Mie scattering curve computed for a particle with radius $a = 10 \mu m$ using an approximation formula from Ref. [4]. It exhibits similar small oscillations as in our data.

The trap by air currents within the enclosure.


