Quantum Conductance

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Our goal was to observe quantum steps of conductance in a gold wire. We observed and measured the quantization of conduction in a thin gold wire before its breaking point. We report conductance values in agreement with the model.
I. INTRODUCTION

Conductance is a measurement of how easily electricity flows through a given material. When the space that the electricity has to flow is reduced to the order of the deBroglie wavelength, quantum properties arise. Quantum conductance in gold wires is specifically important because it is a relatively easy way to show quantum behavior in an undergraduate lab [2] [3]. In addition, quantum conduction is important because of its many applications in nanotechnology because the quantum properties that arise can be understood quantitatively and used for low temperature applications, such as quantum computing [8] [5]. Quantum conduction is also helpful in determining atomic structures and transit properties [7] [1].

Quantum conduction refers to the limit when the constriction radius of a conductive wire, gold in our case, \( w \), stretches to the atomic scale such that \( w \) is much smaller than the length \( L \) of the conductive wire. With respect to the wire dimensions and conductance, there are three models that arise: the classical model, the ballistic model, and the quantum model (Fig. 1). The classical case arises when the length of the wire \( L \) and the width of the bottle-neck are much larger than the mean free path \( l \) of traveling electrons. The ballistic case happens when \( L \) is smaller than \( l \). The quantum case happens when the width \( w \) is on the order of the Fermi wavelength \( \lambda_f \). In our experiment, we will be dealing with the quantum model. Under this condition, conductance becomes quantized, and the system can’t be described with a classical model. Decreasing the radius, \( w \), of our wire means that we are decreasing the number of conduction channels through which electrons can flow. As mentioned before, these conduction channels will decrease in discrete steps because they are quantized. The goal of this experiment is to reduce the radius of our gold wire to the size of one gold atom, allowing only one conduction channel to be in our wire.

II. BACKGROUND THEORY

In the classical model, the conductance of the of the wire, \( G \), is determined by the geometry of the wire

\[
G = \sigma \frac{A}{L}
\]

[2] where \( A \) is the cross-sectional area of the of the wire, \( L \) the length of the wire, and \( \sigma \) is the conductivity of the wire’s material. As the radius of the golden wire becomes smaller, the
conductance is governed by the semi-classical model governed by the Sharvin Conductance formula

\[ G = \frac{2e^2}{h} \left( \frac{k_F \omega}{4} \right)^2, \]  

(2)

where \( k_F \) is the magnitude of the wave vector at Fermi energy, \( w \) is the width of the wire, the factor of 2 comes from the spin degeneracy, and \( \epsilon \) is the energy. The current in our quantum wire can be expressed as

\[ I_k = 2e \int_0^\infty v_k(\epsilon) \left( \frac{1}{hv_k} - 0 \right) d\epsilon = \frac{2e^2}{h} V, \]  

(3)

where \( v_k \) is the Fermi velocity of electrons in a channel \( k \). Using the above formula and Ohm’s Law, the conductance is

\[ G_k = \frac{I_k}{V} = \frac{2e^2}{h}. \]  

(4)

Thus, as the number of channels allowed increases, the conductance is given by

\[ G_n = \frac{2e^2}{h} n, \]  

(5)

where \( n = 1, 2, 3... \) is a positive integer. Another way to understand the quantized conductance is by considering the 1-D infinite potential well model. As the wire gets smaller, the difference between energy levels widens \( (\Delta E_{\text{inf well}} \sim \frac{1}{w^2}) \), and less and less energy levels are allowed below the Fermi energy level. The first five values of conductance, \( G \), are \( G_1 = 7.74809 \times 10^{-5} S \), \( G_2 = 1.54962 \times 10^{-4} S \), \( G_3 = 2.32443 \times 10^{-4} S \), \( G_4 = 3.09924 \times 10^{-4} S \), and \( G_5 = 3.87405 \times 10^{-4} S \).

![FIG. 1. a) The classical case happens when \( L \) and \( w \) are much longer than the mean free path \( l \) of an electron traveling across the wire. b) The ballistic case happens when \( L \) is smaller than the mean free path \( l \). c) The quantum case happens when \( w \) and \( L \) are on the same order as \( \lambda_f \).](image)
III. SETUP

Our system consists of a gold wire glued to a spring sheet by two drops of epoxy. In order to break the gold wire, we secured the spring steel so it bends. We also made a small incision on the goldwire till its radius was about half of its original radius. We then applied a force perpendicular to the center of the spring sheet in order to bend the spring sheet and thus break the wire. We used a USB 6216 DAQ to take measurements with our system. In order to measure the conductance, $G$, we connected two resistors in parallel with the gold wire, $R_{Load}$ and $R_{50}$, where $R_{Load} = 11960.1 \pm 0.1\Omega$ (95% CI), and $R_{50} = 47.239 \pm 0.001\Omega$ (95% CI). We then connected the DAQ in parallel in two places, one across $R_{50}$, and one across both $R_{50}$ and the gold wire sample (Fig 2). We used this unique setup to assure that all the current goes through $R_{50}$ during the measurement. This happens because $R_{50} \ll R_{DAQ}$, where $R_{DAQ} \sim G\Omega$. Because $R_{50}$ is connected in series with the gold wire sample, we use

$$I_{goldwire} = \frac{\Delta V_{R_{50}}}{R_{50}}$$

(6)

to tell us the current across the gold wire sample. We then use

$$G = \frac{1}{R} = \frac{I_{goldwire}}{\Delta V_{goldwire}},$$

(7)

and

$$\Delta V_{goldwire} = \Delta V_{goldwire and R_{50}} - \Delta V_{R_{50}}$$

(8)

to measure $G$.

IV. DATA

We report values of conductance, $G$, of a gold wire. We report $G_1 = (0.7567 \pm 0.0458) \times 10^{-4}S$ (95% CI), $G_2 = (1.3874 \pm 0.2912) \times 10^{-4}S$ (95% CI), $G_3 = (2.3936 \pm 0.1044) \times 10^{-4}S$ (95% CI), $G_4 = (3.2372 \pm 0.1373) \times 10^{-4}S$ (95% CI), $G_5 = (3.8973 \pm 0.0838) \times 10^{-4}S$ (95% CI), all of which are in agreement with the expected values (Fig 3, 4)(Tab 1). We find our $G$ values using the average of the conductance values over a step (Fig 3). We therefore confirm the model and show quantized conductance in a gold wire.
FIG. 2. a) We provide a pressure to the center of the spring sheet in order to bend it and thus extend the gold wire to the point of breaking (see Fig. 1). b) The gold sample is connected in series to a $1.5483 \pm 0.0002$ (95% CI) Volt battery, a $12K$ $\Omega$ rated resistor, and a $50\Omega$ rated resistor. The DAQ unit is connected in parallel twice: once across the $R_{50}$ and once across both the gold sample and $R_{50}$. The resistance of the DAQ is given by $R_{DAQ}$, and we assume $R_{DAQ} >> R_{50}$. We report $R_{Load} = 11960.1 \pm 0.0001\Omega$ (95% CI) and $R_{50} = 47.239 \pm 0.001(95\% \text{ CI})$.

<table>
<thead>
<tr>
<th>N Value</th>
<th>Expected Conductance G (S)</th>
<th>Measured Conductance G (S) (95% CI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$0.774809 \times 10^{-4}$</td>
<td>$(0.7567 \pm 0.0458) \times 10^{-4}$</td>
</tr>
<tr>
<td>2</td>
<td>$1.54962 \times 10^{-4}$</td>
<td>$(1.3874 \pm 0.2912) \times 10^{-4}$</td>
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</tr>
<tr>
<td>5</td>
<td>$3.87405 \times 10^{-4}$</td>
<td>$(3.8973 \pm 0.0838) \times 10^{-4}$</td>
</tr>
</tbody>
</table>

TABLE I. We report conductance values, $G$, that match with the model.

V. CONCLUSION

Overall, when a wire is constricted down to $\lambda_F$, the Fermi wavelength of an electron, quantized steps of conductance become apparent. We model the conductance of a gold wire using the Sharvin Conductance formula, and report measurements that are in agreement with the model. Looking forward, quantized conductance can be seen in modern day exper-
FIG. 3. We report $G$ in $S$ on the y-axis with respect to the length of the gold wire, which is quantitatively known. The expected values are also shown on the graph. We show agreement with the model.

FIG. 4. We report $G$ in $mS$. We show agreement between our measured values (blue dots with error bars) and the expected values of $G_n$, which are given by the horizontal lines.
iments with determining atomic structures and transit properties [7][1], quantum computing [8], and tunneling experiments [4].

VI. ACKNOWLEDGEMENTS

We would like to thank Jonathan Daron, Eric Need, and Jia Qi for their help in the setup of the experiment and for their previous work on the quantum conduction of a gold wire [6].