Quantized Conductance in a Thin Gold Wire

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We observed and measured the quantization of conduction in a thin gold wire. To derive our model for the quantized conductance values we used the Sharvin Conductance formula. We found that for the n=1, n=2, n=3, and n=4 values, the conductance values measured were: 

\[(78.80 \pm .89) \times 10^{-6} \, \Omega^{-1}\], 
\[(150.8 \pm 6.1) \times 10^{-6} \, \Omega^{-1}\], 
\[(204.8 \pm 2.9) \times 10^{-6} \, \Omega^{-1}\], and 
\[(289.1 \pm 8.6) \times 10^{-6} \, \Omega^{-1}\] respectively.
We observed and measured the quantization of conductance in a thin gold wire. Classically conductance is a measure of how well a conductor can allow electricity to flow through it. The electrical and mechanical properties of a piece of a metal are the same for metals of all shapes and sizes. However, when its size approaches the atomic scale all of these general properties break down\cite{1}. No longer is conductance a continuous quantity. Rather, it becomes discrete based off the very nature of its origins. Since conductance is a measure of how well the electrons can flow through a conductor it makes sense that this property becomes restricted when the conductor takes on an atomic scale. Our research is based off this idea that the conductance will change when a conductor reaches an atomic scale. We found that stretching a wire as R. Tolley, A. Silvidi, C. Litte, and K. F. Eid discuss in their paper\cite{2} is one way to create an atomic scale conductor to use in measuring this effect. The procedures to perform these measurements are presented in our research and in their paper\cite{2} as well as the paper by E.L. Foley, D. Candela, K.M. Martini, and M. Tuominen\cite{3}.

![FIG. 1. These are the modes of the cross-sectional views of wire for the three possible situations regarding conductance. (a) The classical wire, where the length $L$ and width $w$ of the bottle-neck part is much larger than the mean-free-path $l$ of traveling electrons; (b) The semi-classical wire, where the length $L$ is smaller than $l$; (c) The quantum wire, where the width $w$ is on the order of the Fermi wavelength $\lambda_F$.](image)

The theory behind the idea of quantized conductance is that the quantum physics dominates the behaviour of a system when the scale of the system becomes atomic. In classical conductance, the width and length of the wire is much larger than the wavelength of an electrons’ motion (See FIG. 1(a)). In this case an electron will scatter several times while traveling through the bottle-neck portion of the wire. The electrons travel is due to a force caused by an electric field from the potential difference across the wire. In that case, the
conductance, $G$, of the wire is determined by the geometry of that wire,

$$ G = \sigma A / L, $$  

where $A$ is the cross-sectional area of the wire, $L$ is the effective length of the bottle-neck portion of the wire, and $\sigma$ is the conductivity of the wire’s material – which remains constant in our experiment.

As the scale of the wire becomes smaller, the conductance begins to behave like the semi-classical model (See FIG. 1(b)). In this model, the length of the wire is smaller than the mean-free-path of the electron (the average distance an electron travels before scattering). As a result, most of the electrons will travel through the wire without scattering. Conductance under that condition is described by the Sharvin Conductance formula,

$$ G = \frac{2e^2}{h} \left( \frac{k_F \omega}{4} \right)^2, $$  

where $k_F$ is the magnitude of the wave vector at Fermi energy [2].

As the width of the wire decreases to the atomic scale the conductance of the wire will become quantized (See FIG. 1(c)). On the atomic scale the length and width, of the bottle-neck portion of the wire, become comparable to the wavelength of the electron. Because of this conducting free electrons are treated as waves of specific wavelengths, instead of particles, on the atomic scale. A good analogue for this system is a one dimensional infinite square well of width $w$. So, the possible allowed wavelengths for an electron are

$$ \lambda_n = \frac{2\omega}{n}, $$  

where $n = 1, 2, 3, 4, ..., \alpha$ and $\alpha$ is any positive integer. Then we assume that all states below the Fermi energy, $\epsilon_F$, are occupied and all states above are empty. So, the shortest wavelength for an electron to occupy is

$$ \lambda_f = \frac{h}{\sqrt{2m\epsilon_F}}. $$  

Since electrons are fermions, only one electron is allowed to occupy each state with a given wavelength $\lambda_n$. Thus, the maximum number of allowed wavelengths is

$$ n = \frac{2w}{\lambda_f}. $$
where $\lambda_F$ is the wavelength of the highest allowed energy state, the Fermi Energy. This means if the width of the wire is half of the Fermi wavelength then only one wavelength is allowed in that state and only one electron is allowed to pass.

When a potential difference is applied across the wire, the current for each conducting channel (or wavelength) is

$$I_k = 2e \int_{0}^{\infty} v_k(\epsilon)[\rho_{kL}(\epsilon) - \rho_{kR}(\epsilon)]d\epsilon,$$

where $v_k$ is Fermi velocity for this wavelength, the factor of 2 comes from two spin states, $\epsilon$ is the energy, and $L$ and $R$ label the right and left sides of the wire. In Eq. (6) $\rho$ is the density of states given by

$$\rho = \sqrt{m/2h^2\epsilon},$$

for $\epsilon > \epsilon_F$ [2]. The current is therefore,

$$I_k = 2e \int_{0}^{\infty} v_k(\epsilon)(\frac{1}{hv_k} - 0)d\epsilon = 2\frac{e^2}{h}V.$$

Then according to Eq. (8) and Ohm’s Law, $V = IR$ (where $R = \frac{1}{G}$), the conductance is,

$$G_k = \frac{I_k}{V} = \frac{2e^2}{h}.$$

This value is independent of the geometry of the wire. As we increase the number of wavelengths allowed, the conductance increases to

$$G = \sum_k G_k = \frac{2e^2}{h}n,$$

where $n = 1, 2, 3, ..., \alpha$ and $\alpha$ is any positive integer. By Eq. (10) the conductance of the wire will change by a constant value as we change the width of the wire. This model does not include potential effects due to quantum tunneling. However, in our research we did not investigate this[4].

Once we had our model (Eq. 9 and Eq. 10), we needed a way to find values for the conductance of our gold wire samples. We began by measuring the potential difference across the gold wire samples. To do this we used our apparatus to stretch the gold wire so that it became increasingly thin. We then analyzed the data to find the voltage values of each step. From there we were able to calculate the conductance associated with each step using Eq. 9. We also calculated the expected values of the conductance for each step using Eq. 10.
To construct our apparatus we looked at a couple designs and decided to model our apparatus after the one developed by R. Tolley, A. Silvidi, C. Litte, and K. F. Eid[2]. The design we used for our apparatus can be seen in FIG. 2 along with a skematic of the gold wire samples we used. The plastic housing was designed to fit a micrometer head into it, which we used to push on the center of our sample so we could stretch the wire. The micrometer head had a large wheel attached to the end of it so that we could turn the micrometer head a small amount more easily. To investigate this, assume that the most precise turn we can make by hand, with a circular wheel of radius \( r = 10.0\text{cm} \) (See FIG. 2), is approximately \( \Delta S_{\text{ideal}} = 0.25\text{cm} \), where \( \Delta S \) is the distance a point on the wheel travels at the edge. Then by \( \Delta S = \Theta r \) we see that we obtain a rotation of \( \Theta = 0.025\text{radians} \) (approximately

FIG. 2. Our apparatus that we used to perform our measurements. The micrometer head was inserted into the back of the main apparatus piece. The set screw was tightened so that the force of the sample pushing on the micrometer would not move the micrometer. The wheel was placed on the opposite end of the micrometer. The micrometer head bends the sample, causing the gold wire to stretch.
1.43 Degrees). To get this same rotation with the bare micrometer head, with a radius of \( r = 0.8 \text{cm} \) (See FIG. 2), we would have to turn the wheel a distance of \( \Delta S = 0.02 \text{cm} \). This distance is much smaller than \( \Delta S_{\text{ideal}} \). So, the wheel was necessary because the samples were very sensitive to even the slightest movement and it allowed us to very finely turn the micrometer head. A wheel with a larger radius would lend itself to more precise rotations of the micrometer head. The plastic housing was secured to a large metal track so that it would not move while we were using it. As the sample bends, the gold wire is stretched to the point where only a few atoms are touching. This allows only a few electrons to travel at a time. This is, in short, quantized conductance[1].

After we had our samples and apparatus completed we set up a circuit to perform a fourwire measurement. This measurement occurred on a circuit we made driven by a 1.5V D-cell battery (Which we measured to have \( V = 1.4852 \pm 0.0010 \text{ Volts} \)) in series with a 100 k\( \Omega \) resistor and our thin gold wire. Using this information, we found that the current we put through the wire was \( I = (14.850 \pm 0.010) \mu\text{A} \). We were then able to measure the voltage across the gold wire using a four wire measurement.

To collect data we ran our Lab View program so that it would take a continuous reading of the voltage across the wire. While measuring the voltage we would turn the wheel until we reached the point where the gold wire would break. The wheel was backed off so that the wire was just barely reconnected. We collected a burst of data while we were then turning the wheel to separate the wire again. The program was set up so that when we instructed it to, it would collect 200,000 data points at a rate of 200 kHz and export the data after each run. The data was then analyzed to determine the pertinent information.

After collecting data, we noticed that in our trial the step like pattern predicted was very evident; this is shown in FIG. 4. After completing the analysis of the data for our trial, we found the conductance and their uncertainties for each step and they are listed in TABLE I.

Once we finished collecting data, we were able to select the smallest range of the data that provided the window of values from a differential of zero to one and a half. Doing so yielded a graph that looked like FIG. 4. The graph output was much clearer and easier to interpret than it’s raw counter part from LabView. These graphs were put into a format that allowed us to manipulate the section of the data that was graphed. This enabled us to pick out the sections of data that were clearly associated with each step. Using this range of data points we were then able to find the average value for a specific step. Likewise, to find
FIG. 3. This figure is a simple wiring diagram for the circuit we used to conduct the experiment. It consists of a 1.5V D-cell battery (Which we measured to have $V = 1.4852 \pm 0.0010$ Volts) connected in series with a resistor of resistance $R=100\,\text{k}\Omega$ and our gold wire sample. Using this, we found that the current we put through the wire was $I=(14.850 \pm 0.010)\,\mu\text{A}$. Also depicted is the DAQ connected in parallel with our gold wire sample to conduct a four wire measurement.

A step's uncertainty we found the standard deviation and multiplied by the t-distribution value and divided by the square root of the number of data points considered. These values can all be found in TABLE I and are graphed in FIG. 5 with their error bars and expected values.

<table>
<thead>
<tr>
<th>$n$ Value</th>
<th>$G_{\text{Measured}}$ (mS)</th>
<th>$u(G)$ (mS)</th>
<th>$G_{\text{Expected}}$ (mS)</th>
<th>Z-Scores</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.07880</td>
<td>0.00089</td>
<td>0.08748</td>
<td>19.4284</td>
</tr>
<tr>
<td>2</td>
<td>0.1508</td>
<td>0.0061</td>
<td>0.1649</td>
<td>4.6382</td>
</tr>
<tr>
<td>3</td>
<td>0.2048</td>
<td>0.0029</td>
<td>0.2424</td>
<td>25.7023</td>
</tr>
<tr>
<td>4</td>
<td>0.2891</td>
<td>0.0086</td>
<td>0.3199</td>
<td>7.1402</td>
</tr>
</tbody>
</table>

Now, comparing our measured values with the expected values from the theoretical model (See TABLE I) we find that all our values are smaller than the expected ones and not in agreement. This can be seen in FIG. 5. The Z-score in the table of our data (TABLE I) is a
FIG. 4. This graph is of the data collected from run 18, using sample 6. Several steps are visible in the data. The steps have values of $(1.4852\pm 0.0010)\text{Volts}$, $(0.1883\pm 0.0021)\text{Volts}$, $(0.0992\pm 0.0041)\text{Volts}$, and $(0.0516\pm 0.0016)\text{Volts}$ for $n = 0$, $n = 1$, $n = 2$, $n = 3$, and $n = 4$ respectively. (NOTE: The $n = 0$ voltage value corresponds to the voltage measurement of the battery we used.)

measure of how far off from the expected value our measurement was. It gives the number of 68% Confident Intervals that lie between the two values.

This systematic deviation in our results is very likely from the amount of noise we received in our measurements due to having the lights on in the room while collecting data. We figured this out after we had collected the majority of our data and performed our analysis. The effect that the lights had on our experiment can be easily seen in FIG. 6 where we collected some data of the voltage with the lights on and then turned them off. Another potential source of noise in our data can also be seen in FIG. 6. On the right hand side of FIG. 6 ($t > 0.24$ seconds) and in FIG. 7 we see two more distinct systematic deviations in our measurements. Looking at these we see that they appear to be periodic. So, looking at the frequencies of these deviations we see that they come out to be about 30Hz and 70Hz. These frequencies are approximately the same as the frequencies of the electromagnetic signal given off by the wifi and the wall outlets in the area. There could also be some ran-
FIG. 5. This is a graph of our findings from sample 6 trial 18. The data was imported into a data analysis program for analysis. The steps are represented here by a data point and its error bars. Each horizontal line represents the expected values for each step. The expected conductance values for $n = 1$, $n = 2$, $n = 3$, and $n = 4$ are $0.08748$ miliSiemen, $0.1649$ miliSiemen, $0.2424$ miliSiemen, and $0.3199$ miliSiemen respectively. It can be seen in the graph that our measured values do not agree with the expected values. The conductance values we found to be $(0.07880 \pm 0.00089)$ miliSiemen, $(0.1508 \pm 0.0061)$ miliSiemen, $(0.2048 \pm 0.0029)$ miliSiemen, and $(0.2891 \pm 0.0086)$ miliSiemen for $n = 1$, $n = 2$, $n = 3$, and $n = 4$ respectively.
FIG. 6. This is a graph of our circuit's voltage over time where we collected data for the first 0.24 seconds with the lights on and then turned them off. Here we can see that turning the lights off would reduce the noise in our measurements significantly.

dom noise due to thermal vibrational energies of the gold atoms[5]. One way of eliminating the possibility of having noise in our measurement due to vibrational thermal energies is to drastically reduce the temperature of the gold wire while taking data[5].

In this experiment we have confirmed the quantization of conductance in a thin gold wire. Using a purpose built apparatus we were able to show that there are discrete steps in the conductance of the gold wire. We were unable to completely remove the noise from our system so there is room for further refinement of our set up. To reduce noise in future measurements we recommend that all circuit and apparatus elements be in complete darkness. Also, being able to shut off wall outlets, wifi, and other potential sources of an electromagnetic signal may help in reducing noise. Some possible refinement to our apparatus would be to integrate a piezo stack into the set up on the micrometer head. This would allow the experimenters to time the firing of the piezo with the collection of data, thus removing one component of human error and removing the need to select the range of data points to
FIG. 7. This is a graph of our circuits voltage over time with Sample 2 where we collected data with the lights off. Here we can see that even with the lights off we still have some systematic noise in our measurements. There are two separate signals seen here, one creating the peaks with large amplitude and high frequency and the other creating the ones with the smaller amplitude and lower frequency. If data was collected in an environment free of live electrical outlets, wifi, and other electronic devices we believe this would eliminate the systematic noise seen here and in turn dramatically improve our results.

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