

## The Projectile Velocity of an Air Cannon

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Pressurized air cannons are inexpensive, safe, and impressive, therefore making them excellent for studying projectile motion in an undergraduate laboratory. However, in order to study projectile motion, the exit velocity of the projectile from the air cannon must be known, and this exit velocity is a function of the internal ballistics of the cannon. Several theoretical models for the internal ballistics of an air cannon have been proposed, but experimental data on exit velocity as a function of initial conditions have been sparse. We have constructed an air cannon, and we provide experimental data in stark disagreement with the existing models along with a new model to explain them.

Cannons have been around for over a millennium and many attempts have been made to correctly describe the trajectory of their projectiles. Cannons are powered by gas expansion in the barrel which causes the projectile to be thrown forward by the force of the expanding gas and play a significant role in illustrating various physics properties such as recoil [1], conservation of momentum [1], the work-kinetic energy theorem [2], and air drag [3], among others. These effects can be illustrated with a relatively simple air cannon. The use of an air cannon as opposed to a rifle or other firearm has several advantages. First, it creates a safer lab environment. But also, the internal dynamics of an air cannon are much simpler and easier to understand because they do not rely on gas expansion driven by an unpredictable explosive mechanism. The dynamics of the projectile's motion after it leaves the barrel of the air cannon are well understood; however, these calculations rely on an accurate knowledge of the initial conditions of the system. This experiment seeks to understand the conditions that give the exit velocity.

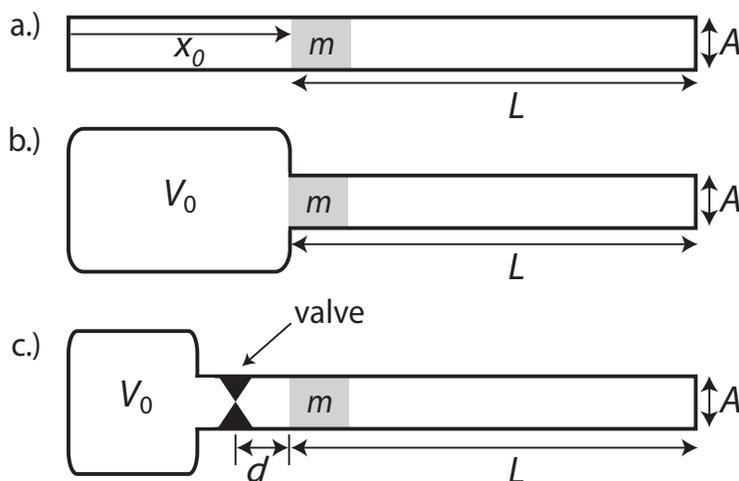


FIG. 1. Three models of the expansion of gas in an air cannon. (a) An explosive gas expands adiabatically [4]. This model is described by Eq. (3). (b) Gas from a pressurized reservoir expands isothermally [5]. See Eq. (5). (c) The expansion of the gas is limited by a valve with a finite flow factor. This is the model advocated by this paper.

The basic concept behind an air cannon is that a reservoir of volume  $V_0$  is pressurized at  $P_0$ . This reservoir is connected to a long barrel of cross-sectional area  $A$  loaded with a projectile of mass  $m$ . As gas in the reservoir expands, the pressure accelerates the projectile

along the length  $L$  of the barrel at which point the projectile exits the cannon at a speed  $v$ .

We would like to find the exit velocity  $v$  as a function of these initial conditions. To do so, two models have been previously proposed. In the first, illustrated in Fig. 1(a), the air from the reservoir (of volume  $V_0 = Ax_0$ ) expands quasistatically and adiabatically [4]. Assuming no air leakage, there are three forces on the slug (illustrated in Fig. 1). There is a force  $AP(x)$  due to the pressure from the gas, an opposing force  $AP_{\text{atm}}$  from the atmosphere, and a linear friction force  $f$ . By Newton's Second Law, these add up to give

$$F = m \frac{dv}{dt} = mv \frac{dv}{dx} = AP(x) - AP_{\text{atm}} - f, \quad (1)$$

where  $P_{\text{atm}}$  is atmospheric pressure and  $f$  is a term of constant friction. Since we are assuming adiabatic expansion, we know that  $P(x)(V_0 + Ax)^\gamma = P_0V_0^\gamma$ , where  $\gamma = 7/5$  for diatomic gasses such as air. Thus, in the adiabatic case, we have

$$mv \frac{dv}{dx} = A \left( \frac{P_0V_0^{7/5}}{(V_0 + Ax)^{7/5}} - P_{\text{atm}} \right) - f, \quad (2)$$

which yields an exit velocity at  $x = L$  of

$$v = \sqrt{\frac{5}{m} \left( P_0V_0 \left( 1 - \left( \frac{V_0}{AL + V_0} \right)^{2/5} \right) - 2ALP_{\text{atm}} - 2Lf \right)}. \quad (3)$$

The second model, illustrated in Fig. 1(b) takes the expansion of gas to be quasistatic and isothermal. Following the derivation given in ref. [5], we know that  $P(x)(V_0 + Ax) = P_0V_0$ . Plugging into Eq. (1) gives

$$mv \frac{dv}{dx} = A \left( \frac{P_0V_0}{V_0 + Ax} - P_{\text{atm}} \right) - f. \quad (4)$$

or

$$v = \sqrt{\frac{2}{m} \left( P_0V_0 \ln \left( 1 + \frac{AL}{V_0} \right) - ALP_{\text{atm}} - Lf \right)}. \quad (5)$$

Note that this equation differs slightly from the result offered by ref. [5]. This is because we are not making the simplifying assumption that  $P_0 \gg P_{\text{atm}}$ .

We find that in both of these equations, the frictional term  $f$  serves as simply a horizontal offset for the model. Because we find that the application of a very small  $P_0 - P_{\text{atm}}$  ejects the slug and because we see from our data in Fig. 3 that our data are not horizontally offset, we take  $f \approx 0$ .

A real pneumatic air cannon must have a valve between the reservoir and the barrel in order to allow pressurization of the reservoir before the firing of the projectile. While one could in theory imagine a perfect valve that does not have any appreciable effect on the air that flows past it, this is in practice hard to realize. A reasonable valve type is a solenoid-actuated diaphragm valve, which has an associated flow coefficient  $C_v$ . Such valves have been used on pneumatic cannons on *Mythbusters* and the Punkin Chunkin' contest [6]. The flow rate  $Q$  through a valve with coefficient  $C_v$  is a function of the pressure drop across the valve [7]. Therefore, it is unreasonable to ignore the effect of the valve, since the pressures on either side of the valve are not the same.

We propose a new model, shown in Fig. 1(c), that takes into account the flow rate through the valve. The flow rate  $\tilde{Q}$  (a quantity given in units of molecules/s differentiated from  $Q$  which is in L/min at STP) is a function of the pressure in the reservoir  $P(t)$  and the pressure in the barrel  $P_b(t)$ . It is given to be [7]

$$\tilde{Q}(t) = \begin{cases} (.471)BC_vP(t)\sqrt{1/G_gT}, & P(t) \gtrsim 2P_b(t), \\ BC_vP(t)\left(1 - \frac{2}{3}\left(1 - \frac{P_b(t)}{P(t)}\right)\right)\sqrt{\left(1 - \frac{P_b(t)}{P(t)}\right)/G_gT}, & P(t) \lesssim 2P_b(t), \end{cases} \quad (6)$$

where  $G_g = 1$  is the specific gravity of air,  $T$  is the temperature in the reservoir, and  $B$  is a proportionality constant to convert between  $Q$  and  $\tilde{Q}$ . We are assuming that temperature is constant, which is an approximately valid assumption given our experimental data (we only measure a  $\sim 1$  K temperature drop in firing the cannon).

Using the Ideal Gas Law, the gas inside the tank and outside the tank is characterized by the equations

$$P(t)V_0 = N(t)k_B T, \quad (7)$$

$$P_b(t)A(d + x(t)) = N_b(t)k_B T. \quad (8)$$

The number of molecules in the tank and barrel are governed by the flow of molecules between them through the valve,  $dN/dt = -\tilde{Q}$  and  $dN_b/dt = \tilde{Q}$ , and, by Newton's second law, the position function, as above, is governed by  $d^2x/dt^2 = A(P_b(t) - P_{\text{atm}})/m$ . These differential equations, when combined with the initial conditions, can be numerically solved to give  $v = dx/dt$  as a function of initial pressure  $P_0$ .

We use an ASME-Code Horizontal Pressure Tank with a volume of  $V_0 = 4.196 \pm .010$  L (all measurements given to a 95% confidence interval). We attach a pressure sensor (Omegadyne

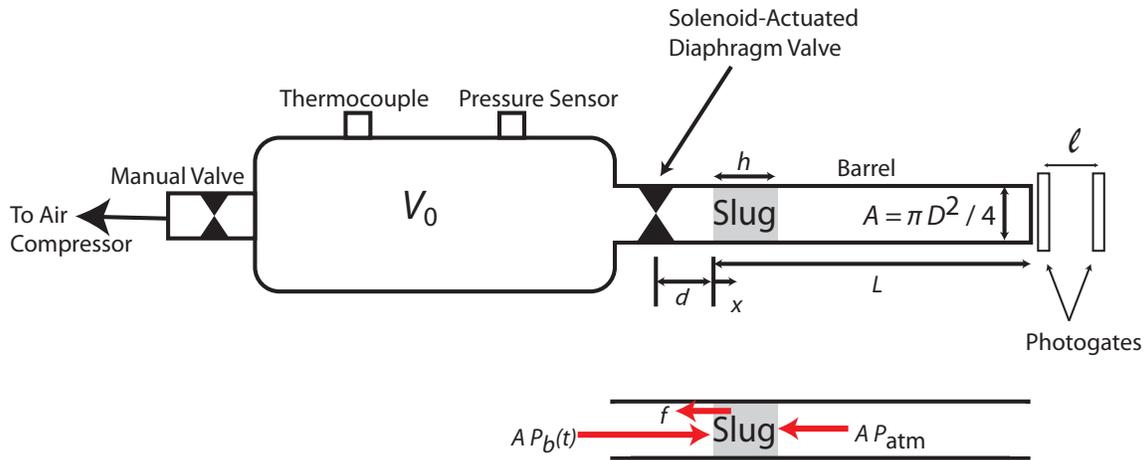


FIG. 2. A schematic of the air cannon. The tank is a reservoir with a volume  $V_0 = 4.196 \pm .010$  L, initially charged to pressure  $P_0$ . The tank is discharged using a diaphragm valve with  $C_v = 2.80$ . The pressurized air then propels the slug, with a height of  $h = 2.39293 \pm .00058$  cm, a distance of  $L = 88.34 \pm .08$  cm out of the barrel of cross sectional area of  $A = 2.872326 \pm .000026$  cm<sup>2</sup>.

Model PX309-100GV), thermocouple (Omega Model TC-K-NPT-E-72), solenoid-actuated diaphragm valve (Granzow Model 21HN5KY160-14W) with  $C_v = 2.80$ , and air intake hand valve to the tank as shown in Fig. 2. The diaphragm valve opens when a current of 440 mA activates a solenoid in the valve. This air rushes into the attached barrel—a 1.905-cm diameter, 91.44-cm long 304/304L seamless stainless steel threaded pipe—and forces the slug out. The exit velocity is measured by two photogates positioned  $\ell = 24.6 \pm 0.9$  cm apart. We collect data simultaneously from the thermocouple, pressure sensor, and photogates at a sampling rate of 10 kHz. The photogate trigger has an internal clock of 100 kHz. Data acquisition is triggered when an ammeter, connected to the solenoid actuator circuit, reads an increasing current across 5 mA.

We loaded the cannon with an acetal copolymer cylindrical plastic slug of mass  $m = 9.600 \pm .048$  g, height  $h = 2.39293 \pm .00058$  cm, and diameter  $D = 1.91237 \pm .00058$  cm. The diameter of the slug was such that it just fit into the barrel of the cannon. We tested whether air could escape from around the edges of the slug by closing the diaphragm valve and attempting to load the cannon. The slug was sufficiently air tight that it built back pressure when we tried to insert it.

Our loading procedure is to open both the diaphragm valve and the hand valve to make sure that there is not a pressure built up between the slug and the tank. Then, we place the slug in the barrel and use a steel rod to slowly push the slug back a predetermined length  $L - h = 85.95 \pm .08$  cm. We then close the solenoid-actuated diaphragm valve before removing the rod. We use an air compressor to pressurize the tank to the desired initial pressure  $P_0$ , close the hand valve, and wait as long as two and a half minutes until the reading on the digital pressure gauge stabilizes. After this, we fire by opening the diaphragm valve.

Using this loading procedure, we collected data for the exit velocity  $v$  of the slug as a function of initial reservoir pressure  $P_0$ . These data are shown in Fig. 3(c). We also did the same thing for a slug of approximately twice the length and mass (specifically,  $m = 19.400 \pm .048$  g and  $h = 4.80187 \pm .00058$ ) as shown in Fig. 3(a) and an aluminum slug ( $m = 19.900 \pm .048$  g and  $h = 2.59232 \pm .00058$  cm) as shown in Fig. 3(b). It is clear that our data are in gross disagreement with both the adiabatic and isothermal models. However, we are in much better agreement with a numerical calculation of the valve flow model presented above. However, we would expect our model to be an upper bound, given that we are not taking into account the higher-order dissipative effects of turbulent flow and air drag in the cannon. One such way to incorporate drag may have been proposed by ref. [8], in which the authors consider the fact that the column of air outside the slug needs to be accelerated in addition to the slug itself. The first-order linear friction term has also not been included in our model. However, as discussed above, we only expect this to introduce a horizontal offset that our data do not show.

We also took data for different initial positions  $L$  of the projectile in the barrel. These data are shown in Fig. 4. Our model is given credence by the fact that it approximately qualitatively matches our data. We believe that the discrepancy comes from the fact that we are not taking into account the effects of turbulent flow or pressure wave propagation time.

In conclusion, we have shown that both the adiabatic and isothermal expansion models are not consistent with the material limits of a real air cannon fired by means of opening a valve. This is because the assumption undergirding both models that the air pressure in the reservoir is the same as the pressure in the barrel is difficult to meet given the necessity of a valve with associated flow coefficient  $C_v$ . We put forward an alternative model that still serves as an upper bound for exit velocity of the slug that is much closer to the experimental

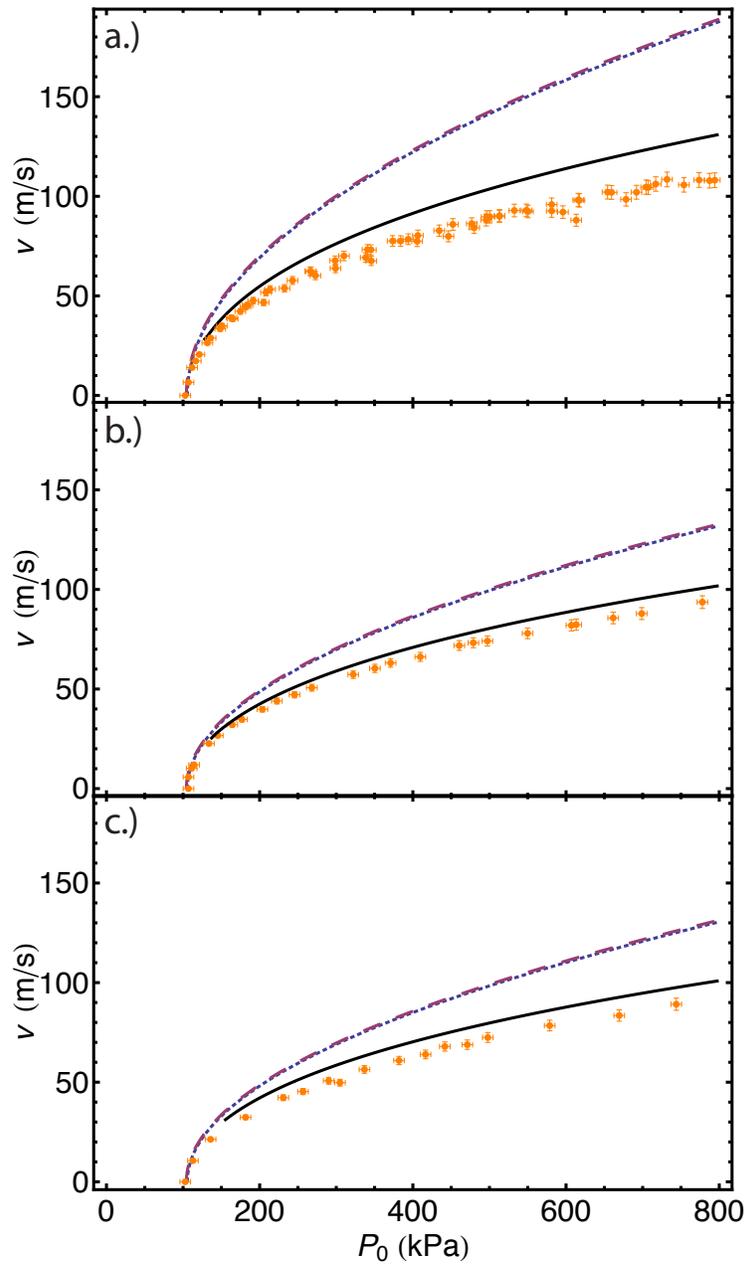


FIG. 3. (color online.) Exit velocity as a function of initial reservoir pressure of (a) a plastic slug with  $m = 9.600 \pm .048$  g and  $h = 2.39293 \pm .00058$ , (b) a plastic slug with  $m = 19.400 \pm .048$  g and  $h = 4.80187 \pm .00058$ , and (c) an aluminum slug with  $m = 19.900 \pm .048$  g and  $h = 2.59232 \pm .00058$  cm. The pink curve is the isothermal model and the blue curve is the adiabatic model. The two models are close to each other because the temperature drop associated with the adiabatic expansion is so small. Our data disagree with both. The orange curve is a numerical plot of our new model, which provides a much more reasonable upper bound to our data.

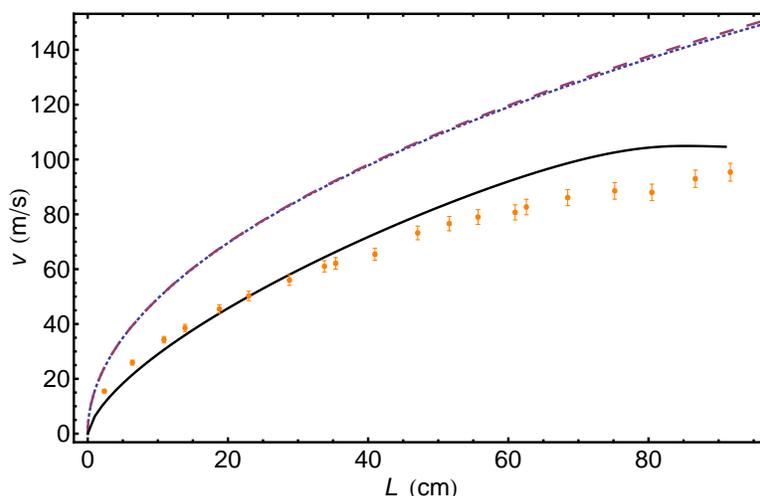


FIG. 4. Exit velocity as a function of initial position  $L$  of a slug in the barrel. This slug has  $m = 9.600 \pm .048$  g and  $h = 2.39293 \pm .00058$ . The pink curve is the isothermal model and the blue curve is the adiabatic model. The two models are close to each other because the temperature drop associated with the adiabatic expansion is so small. Our data disagree with both. The orange curve, which is in approximate qualitative agreement with our data.

data. In the future we hope to use this model to accurately predict the final position of a projectile fired from an air cannon.

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- [5] M. Denny. "Internal Ballistics of an Air Gun", *The Phys. Teacher*. **49** 81-83 (2011).
- [6] Solenoid-actuated diaphragm valves have been used in air cannons by, for example, Discovery Channel's *Mythbusters* in Episode 61 (summarized on the unaffiliated web page [http://kwc.org/mythbusters/2006/09/episode\\_61\\_deadly\\_straw\\_primar.html](http://kwc.org/mythbusters/2006/09/episode_61_deadly_straw_primar.html)).

Also, the more efficient butterfly valves have been used in construction of air cannons in the Punkin Chunkin World Championship (see <http://www.popsci.com/gadgets/article/2010-01/air-cannon-sends-pumpkins-3700-feet>).

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