Holography and Optical Vortices

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Optical vortices have possible applications in the emerging field of quantum computing and in optical manipulation. The creation of optical vortices can be achieved through use of spiral phase plates and holography. We utilized simple holographic techniques to successfully construct optical vortices of varying modes in an undergraduate lab. By improving these techniques, we hope to eventually be able to dynamically manipulate holographic images using spatial light modulators.
Optical vortices have possible applications in the emerging field of quantum computing because of their ability to store qubits in their orbital momentum states [6] and in optical manipulation through the harnessing of thermal forces [5]. In order to investigate these properties, it is possible to create optical vortices in the beam of a Helium-Neon laser through the use of spiral phase plates [1, 3] and, more interestingly, through holographic techniques [1, 3, 6]. Moreover, such holographic techniques are becoming easier to utilize in the undergraduate laboratory [2, 4, 7].

An optical vortex is characterized by a screw-shaped topological wavefront distribution, as shown in Fig. 1, which results in a planar image of a doughnut-shaped intensity distribution, [1]. The phase of the wavefront around its translational axis varies linearly with the azimuthal angle in phase with the regular longitudinal oscillation and with an integer multiple frequency of that oscillation frequency.

This rotation of the wavefronts can be used to impart a force on objects trapped within the beam profile that moves them towards the beam’s center, while the phase difference of rotational motion induces an orbital angular momentum within each individual photon [8]. This angular momentum of the optical vortex can be used to apply torques to objects trapped within the beam.

The special case of an optical vortex stems from the complex wave equation that dictates the form of a laser[1]. Any form of laser light is a particular solution, $U$, to the complex wave equation of a given integer mode,

$$\nabla^2 U - \frac{1}{c^2} \frac{d^2 U}{dt^2} = 0.$$  

(1)

To solve the complex wave equation, we expect a laser intensity profile to follow a Gaussian curve, with peak intensity in the center of the circular beam profile. Adapting the form of the Gaussian beam profile, with an optical vortex, the exponential basis of the Gaussian beam is maintained, but extra terms are introduced and manipulated within the solution to accommodate the vacancy of intensity within the beam profile. So for a Gaussian intensity profile, a wave-front radius $R(z)$, and beam radius $w$, the accepted field equation is,

$$U_{\text{Gaussian}}(r, z) = U_0 \frac{w_0}{w(z)} \exp \left( -\frac{r^2}{w^2(z)} \right) \exp \left( -ikz - ik\frac{r^2}{2R(z)} + i\psi(z) \right).$$  

(2)

along the translational $z$-axis and radial $r$-axis, with wave number $k$, where $\psi$ is the Gouy phase constant that adds to the phase of the beam.
By introducing the phase variable of the helical wavefront, the solution specific to the optical vortex is similar to the Gaussian wave equation, but the critical addition to the equation is the complex exponential term dependent upon the phase change, \( \phi \). The solution to the optical vortex complex wave equation is

\[
U_p^\ell(r, \phi, z) = U_0 \exp \left( -\frac{ikr^2}{2R} - \frac{r^2}{2w^2} - i(2p + \ell + 1)\psi(z) \right) e^{-i\ell\phi} (-1)^p \left( \frac{r^2}{w^2} \right)^{\ell/2} L_p^\ell \left( \frac{2r^2}{w^2} \right) \tag{3}
\]

where \( L_p^\ell \) is a generalized Laguerre polynomial \([1]\).

Within the solution for the optical vortex, the modular integers of \( \ell \) and \( p \) are critical in determining the shape and propagation of the wave fronts. The value of \( \ell \) contributes to the total phase change across the circumference of the beam, known as the vortex charge. The value of \( p \) corresponds to the number of radial maxima present in the beam, for which we assume \( p = 0 \) because we are experimenting with a single maxima beam.

The critical element of the optical vortex solution to explain its wave-front shape is the exponential term \( e^{-i\ell\phi} \) \([1]\). The angle \( \phi \) will vary between 0 and \( 2\pi \), so the value of \( \ell \) will dictate the number of helical rotations completed as the wave propagates along the z-axis as shown in Fig. 1.

![FIG. 1. For each of the modular waves, the helix completes a full rotation, \( 2\pi \ell \), for each separation between the wavefronts of the beam. All of these images assume a \( p \) modular value of 0, since we are only concerned with the 0th radial maxima. The helix structure of the first three modes are shown in the image above.](image)

Another key component to the rotation of the wavefronts as the wave propagates is the orbital angular momentum associated with optical vortices, which is a product of the changing form of the wave. Because the wave is helical, the Poynting vector associated with the electro-magnetic wave of light will not always be along the translational axis, as is the case with a traditional plane wave. Therefore, the Poynting vector will have an angular
component, contributing to a net orbital angular momentum for the wave, found in the form of the optical vortex.

We used a holographic technique to create this optical vortex. Holography is the study of being able to record and reconstruct images that store not only intensity information, but also relative phase information, relying on the interference of waves. The interference pattern created between a plane-wave reference beam and a scattered beam is recorded on a photoplate as illustrated in Fig. 2. Later, the scattered beam can be reconstructed by shining the reference beam onto the developed photoplate.

In order to construct an optical vortex using holographic techniques, we begin by mathematically simulating the recording of the hologram on a photoplate. If we interfere a plane wave $e^{-ik_xx - ik_zz}$ with Eq. (3) for $p = 0$, the resulting intensity distribution is [1]

$$I = I_0 (2 + 2 \cos (k_x x - \ell \theta)),$$

$$= I_0 \left( 2 + 2 \cos \left( \frac{2\pi}{\Lambda} x - \ell \theta \right) \right),$$

for fringe spacing $\Lambda$. We used Mathematica to create a density plot of this intensity distribution for different charge $\ell$ and printed the plots on overhead transparencies to create diffraction gratings. These density plots are shown in Fig. 3. In order to construct our optical vortex, a plane wave laser is directed through this diffraction grating. The first diffracted order is the vortex.

However, before looking at the optical vortex, we must calibrate our CCD camera. To do this, we use a circular aperture, and analyze the resulting intensity profile using the Fraunhofer diffraction model. We shine the Helium-Neon laser through a 100 micron pinhole $L = 15.52 \pm .10$ cm from the CCD and record the resulting image. (All uncertainties in this paper are given to a 95% confidence interval.) We then use the Java-based ImageJ64 processing software to plot the intensity of the diffraction pattern across the image as shown in Fig. 4. We did this twice. In the first data set, we heavily attenuated the laser so as not to saturate the CCD. However, this lack of intensity came with the consequence of having a minimal first order fringe. Thus, we increased the intensity of the laser incident on the aperture so that we could see the first order fringe. We then threw out all of the data in the saturated region of the CCD camera so that the fit was only to the unsaturated data.

It is well known that in the Fraunhofer model, the angular intensity distribution of light with wave number $k$ having been directed through a circular aperture of radius $a$ is given
FIG. 2. A schematic setup for holography. (a) Recording. A laser beam comes in and is split. One half of the beam scatters off the object and the other half of the beam (the “reference beam”) does not. When the two beams are recombined at a photo plate, the interference pattern is recorded. (b) Play back. By shining the reference beam back through the interference pattern shown on the reference plate, the phase of the light is reconstructed creating a virtual image for an observer standing below the apparatus.

by

\[ I = I_0 \left( \frac{2J_1(ka \sin \theta)}{ka \sin \theta} \right)^2, \]

for the Bessel function \( J_1 \). Now, if we project this angular intensity distribution on a CCD screen a distance \( L \gg a \) away, as shown in Fig. 5, we see that \( \sin \theta \simeq \tan \theta = r/L \). Thus,
FIG. 3. Diffraction gratings generated from the intensity profile of an interference pattern between a plane wave and an optical vortex of charge $\ell$ as given in Eq. (5). Based on the way Mathematica’s \texttt{ArcTan[]} and \texttt{DensityPlot[]} functions work, the actual functions inputted into the program differ slightly from Eq. (5), although their similarities should be evident.

FIG. 4. An intensity distribution for a pinhole being recorded in the Fraunhofer region. The data are fit to Eq. (8) with the result that $A = (2.3877 \pm .0078) \times 10^{-2}$ pix$^{-1}$. A second intensity distribution, in the right corner, for a pinhole being recorded in the Fraunhofer region. This data was taken with a higher intensity beam. The data in the center have been dropped because in this region the CCD camera was saturated. The data that remain have been fit to a single instance of Eq. (8) with $A = (2.4232 \pm .0023) \times 10^{-2}$ pix$^{-1}$. 
our radial intensity distribution on the screen becomes

\[ I = I_0 \left( \frac{2J_1((ka/L)r)}{(ka/L)r} \right)^2, \]

(7)

\[ = I_0 \left( \frac{2J_1(Ar)}{Ar} \right)^2, \]

(8)

for \( A \equiv ka/L \). Fitting this model to our two data sets and then averaging the results gives us that \( A = 2.4055 \pm 0.0092 \) pix\(^{-1}\).

FIG. 5. In order to calibrate the camera, we directed a laser through a pinhole of radius \( a = 50 \) \( \mu \)m and recorded the Fraunhofer diffraction pattern on a CCD camera.

Now, given our aperture-CCD separation of \( L = 15.52 \pm 0.10 \) cm, our precision pinhole of diameter \( 2a = 100 \) \( \mu \)m, and our Helium-Neon laser emitting \( \lambda = 2\pi/k = 632.8 \) nm, we get that \( A = 3199 \pm 21 \) m\(^{-1}\). Combining, therefore, our \( A \) value as measured in pixels on the CCD with this calculated \( A \) value in meters, we get that the calibration scaling factor of our CCD camera is \( 7.520 \pm 0.057 \) \( \mu \)m/pix. This is very close to the manufacturer’s reported pixel separation of 7.4 \( \mu \)m/pix, but not in agreement. We believe that this discrepancy comes from a distortion introduced by QuickTime, the program that we use to record images from the CCD to the computer.

Once we calibrated the CCD camera, we wanted to experimentally test the setup, which we could easily complete by measuring the interference pattern between two plane waves. The expected interference pattern between two plane waves is due to a phase difference in the waves that approach the screen. In this case, we affect the path of the waves by interfering them at a slight angle, so that each ray travels a different distance, resulting in a phase difference when rays from the two waves combine, evident as vertical fringes in the image of their combination. It is well-know that we can model the angular positions of each of these fringes mathematically—for a separation of sources \( d \) and wavelength \( \lambda \) as shown
in Fig. 6—as,

\[ m\lambda = d \sin \theta, \] (9)

for a given numerical maximum away from the center, \( m \).

![Diagram of two plane waves bouncing off a mirror and interfering](image)

**FIG. 6.** Two plane waves with initial separation \( d \) bouncing off of a mirror and then interfering with each other. Since \( L \gg d \), we can say that they are traveling approximately parallel so that the path length difference between the two paths to a point on the CCD screen vertically displaced by \( y \) as drawn in the picture is \( d \sin \theta \).

For our setup, the angle between the waves was extremely small, so we utilize the small angle approximation \( \sin \theta \approx \tan \theta = y/L \). Therefore, the equation for the separation of the fringes, \( \Delta y \) in the interference pattern is,

\[ \Delta y = \frac{\Delta m \lambda L}{d} = \frac{\lambda L}{d}, \] (10)

since each fringe is \( \Delta m = 1 \) apart.

Using the setup shown in Figs. 6 and 7, we record the interference pattern shown in Fig. 8. Using ImageJ software and Mathematica techniques, we analyze the data in terms of pixels, and use our calibration factor to scale the data in terms of microns. From the data, we measure an average separation of \( \Delta y = 184.4 \pm 2.2 \) \( \mu \text{m} \), which agrees with the expected value from the model, \( \Delta y = 173 \pm 10 \) \( \mu \text{m} \).

Now that we are confident in the calibration of our equipment, we can begin to look at optical vortices. When we shine the laser on the grating shown in Fig. 3, we get an optical vortex in the first diffracted order. The evidence for this is twofold. First, when we look at the first diffracted order, we see a hole, as expected and as shown in Fig. 9(a). Also, when we interfere this vortex with a plane wave on the CCD screen, we reconstruct the pattern on
FIG. 7. The setup above was used to interfere the two plane waves and take the image using the CCD camera at the top right of the image. The beam splitter in the center of the setup splits a single beam, which is reflected from the mirrors and sent along the length of the room. The two beams are then reflected from the opposite side of the room, using a mirror that is not pictured, back to the screen of the CCD directly behind the beam splitter where they interfere.

FIG. 8. The interference pattern resulting from the combination of two plane waves on the screen of the CCD camera. The spacing $\Delta y$ between the fringes of the pattern as measured experimentally agreed with the expected values, which verifies our calibration factor and our setup’s accuracy.
the grating as shown in Fig. 9(b). This is as expected, considering that we mathematically interfere a plane wave with a vortex in order to get the diffraction grating.

We have successfully used holographic techniques to create optical vortices. Future work will include interfering these vortices with spherical waves in order to see the appropriate diffraction patterns. Also, we hope to be able to achieve optical manipulation once we have a fuller understanding and more complete control over these vortices. Finally, we hope to use the holographic techniques we have developed here to produce holograms of objects that can be used in undergraduate optics instruction.

![vortices](image)

FIG. 9. (a) When we shine the Helium-Neon laser through the $\ell = 1$ grating, we get the two vortices that are evident in the first diffracted order of the beam. (b) When we interfere one of these vortices with a plane wave, we reconstruct the pattern that was on the grating.


